Abstract—Qualitative evaluation and comparison of changes of indications of objects having different nature is used by designers, managers, people making decisions (PMD) and experts to make the decisions more reasonable. For support of such activity on the analysis of changes of data connected with certain dates and time intervals, models of fuzzy time series are applied. In this article a model of fuzzy tendency the carrier of which is a fuzzy time series and its variety — elementary tendency model — is offered. The offered models are applied for solution of the problem of summarization of fuzzy time series in terms of tendencies.

Keywords Fuzzy Time Series, Fuzzy Tendency, Elementary Tendency, Summarization.

I. INTRODUCTION

In connection with increasing volume and speed of storage of data connected with certain dates and time intervals in data bases, the new technology of data analysis Data Mining for Time Series Data Base (TSDM) is actively developed. Forming these time sequences on the basis of domains permits to consider them in the form of time series (TS). A number of distinctions of such time sequences from traditional (classic) TS, considered in statistic theory of analysis and forecast of TS, can be noted: these time sequences are short TS; such time sequences can be represented not only by numerical but also by linguistic values; it is difficult or impossible to determine suppositions about random nature of values for such time sequences. Distinctive properties of Time Series Data Base generate changes in the aggregate of aims and tasks which form the direction Data Mining for Time Series Data Base[1]: segmentation, cluserisation, classification, indexing, summarization, disclosing anomalies, frequency analysis, forecasting, extraction of associative rules. At present the methodology of solving the indicated problems is formed on the basis of methods and models of intelligent analysis, representation and processing of data [2, 3, 4, 5]. The basis of the offered methods of intelligent data analysis is the concept of the fuzzy time series (FTS) constructed on the base of levels of TS [5, 6] or their first differences [3]. In this article an original model of FTS on the basis of fuzzy tendencies is offered. The offered model describes the behaviour of FTS in the form of the sequence of fuzzy tendencies represented by information granules. Application of information granules for problems of Data Mining for Time Series Data Base is presented in the work [7]. An indisputable advantage of data granulation is the possibility of representation of models of TS at different levels of abstraction in the linguistic form, which permits to widen the accessibility of using models of TS by application users and to improve their interpretability.

In the second part of the article models of a fuzzy time series and a fuzzy tendency are considered. In the third part a special scale for generating an elementary fuzzy tendency is given. Procedures of granulation of FTS in the basis of FT are presented in the fourth part, there is also described the solution of the problem of summarization of TS in terms of FT and presented the experiment on the solution of the problem of summarization of a synthetic TS as the problem of determination of a fuzzy tendency.

II. FUZZY TENDENCY MODEL

A. Concept of linguistic evaluation

One of the problems of the analysis of TS is the analysis of FTS behaviour, that is change of values of TS levels. The solution of the problem of the analysis of TS behaviour expressed in linguistic form can serve as the linguistic evaluation of the behaviour. Linguistic evaluations (LE) are the means of qualitative evaluation and comparison of characteristics and indications of objects having different nature used by designers, managers, people making decisions (PMD), experts. An important property of linguistic evaluations is wide application in practice in making decisions for expressing knowledge about the degree of correspondence of the object being observed or its characteristics to some objective or subjective criterion. The stated property determines the class of absolute LE, which reflects the static aspect of evaluation. The following expressions can be examples of such evaluations: "Satisfactory", "Good", "Bad", "Big", "Small", "Medium", etc. The semantics of absolute linguistic evaluations depends on the context of the environment in which they are used and modeled by fuzzy sets.

Another important property of linguistic evaluations is conditioned on the possibility of ranking them, it permits to present the aggregate of LE in the form of some system with relations. Binary relations formed on the set of absolute LE generate comparative linguistic evaluations by different criteria such as "More", "Less", "Approximately Equal", "Earlier",/
"Later", "Rather", "Better", etc. Comparative evaluations made on the basis of absolute LE can represent changes in different universes: in the universe of objects, in the time universe, in the universe of problems and they express dynamic aspect of evaluation. The semantics of comparative evaluations is also context-dependent and can be modeled on the basis of fuzzy sets.

It is noted in the article [4] that linguistic evaluation has indications expressing the degree of intensity of this evaluation. These indications can be represented in the linguistic form, usually used by people: "Very", "Insignificantly", "Approximately", etc.

Context-dependent linguistic evaluations considered above are given in expert way as a rule, and they are called expert evaluations. In case of impossibility of receiving expert evaluations of indications of objects, abstract linguistic evaluations are used, let us consider such evaluations among the class of context-free linguistic evaluations.

Let us introduce definitions.

B. Model and kinds of a fuzzy tendency

Let us introduce definitions. Let some time series (TS) \( \mathcal{X}_t=\{t_i, x_i\} \) is given, \( i \in [1,n] \), \( n \) – is the quantity of members of the series, \( x_i \in \mathbb{B}, t_i \in \mathbb{B}_t \).

Let us call the ordered sequence of observations over some phenomenon the states of which change in time if the value of the state at the instant \( t_i \) is expressed with the help of the fuzzy label \( \tilde{x}_i \in \tilde{X}_x \), \( i \in [1,n] \), \( n \) – is the quantity of members of series, a fuzzy time series (FTS). That is we represent a fuzzy time series in the form \( \tilde{X}_x=\{t_i, \tilde{x}_i\} \), where \( \tilde{x}_i \) – \( i \)-th fuzzy set (fuzzy label), \( t_i \) – \( i \)-th value of the instant of time, \( t_1 \leq t_i \leq t_n \), \( n \) – is the quantity of members of FTS. Any FTS can be represented in the form of the sequence of fuzzy labels \( \tilde{x}_j=\{w_m, \mu_{\tilde{x}}(w_m) \} \), \( w_m \in w, w \subseteq B \) on the basis of linguistic (context-dependent or context-free) evaluation of levels of TS \( x_i \in \mathbb{B} \).

Let us call the fuzzy label \( \tau_k \in \tilde{X} \) expressing the character of change (systematic motion) of the sequence of fuzzy values of FTS \( \tilde{X}_x \), in the given interval of time the fuzzy tendency (FT) of a fuzzy time series. A fuzzy tendency determines the nature of FTS not in analytic, but in the linguistic form.

Each fuzzy tendency \( \tau_k \) of the fuzzy time series \( \tilde{X}_x \) can be represented by the fuzzy set \( \tau_k =\{\tilde{t}_i, \mu_{\tau}(\tilde{t}_i) \} \), \( i \in [1,n] \) with the function of membership in the fuzzy time series \( \mu_{\tau} (\tilde{t}_i) \), where \( \tilde{t}_i \) is the model of the following form:

\[ \tilde{t}_i = \langle v_t, \alpha_t, \Delta t \rangle, \]

where \( \alpha_t \) is the intensity of the tendency. Let us compare fuzzy labels \( \tau_k \) and base types of tendencies "Increase", "Decrease", "Stability". On the basis of base types derivative types of tendencies, such as "Fluctuations", "Chaos", "Load", "Idle time", etc., can be formed.

A. ACL-scale for generation of the model of the elementary tendency

In this part a special linguistic scale is offered as a tool of both absolute and comparative linguistic evaluation – ACL-scale (Absolute & Comparative Linguistic). This scale will be
applied for construction of the model of the elementary fuzzy
tendency.
The model of ACL-scale $S_e$ for determination of absolute and
comparative linguistic evaluations is representable in the form
of linguistic variable with relations

$$ S_e = \langle \text{Name}_{\_} S_e, \, \tilde{X}, \, B, \, G, \, P, \, \text{TTend}, \, \text{RTend} \rangle, $$

where Name$_{\_} S_e$ – is the name of ACL-scale, $\tilde{X}$ – is the base
term-set of absolute LE (linguistic name of gradations), for
example, $\tilde{X} = \{\text{Bad}, \, \text{Satisfactory}, \, \text{Good}, \, \text{Excellent}, \, \ldots\}$, $\tilde{x}_i \in \tilde{X}, \, B$ – is the universal
set on which the scale is determined, $x \in B$ . $G$ – are syntactic rules of deduction
(generation) of chains of evaluation propositions (derivative of
terms which do not enter into the base term-set), $P$ - are
semantic rules which determine membership functions for
each term (they are usually given in an expert way),

$$ \text{TTend}(\tilde{x}_i, \tilde{x}_j) \, \text{is the linguistic relation fixing the type of}
\text{change between two evaluations } \tilde{x}_i, \, \tilde{x}_j \text{of the scale,}
$$

$$ \text{RTend}(\tilde{x}_i, \tilde{x}_j) \, \text{is the linguistic relation fixing the intensity}
\text{of difference between two evaluations } \tilde{x}_i, \, \tilde{x}_j \text{of the scale.}
$$

The relation $\text{TTend}(\tilde{x}_i, \tilde{x}_j)$ is the fuzzy linguistic relation
which is applied for determination of comparative linguistic evaluation $\nu_0 = \text{TTend}(\tilde{x}_i, \tilde{x}_j)$ which characterizes the
direction of change (increase or decrease) of the value of the absolute LE $\tilde{x}_i$ with respect to $\tilde{x}_j$ which can be represented
by linguistic expressions, for example, by values from the set
\{INCREASE, DECREASE, STABILITY, ZERO\}. Let us note
that each evaluation $\nu_0 = \text{TTend}(\tilde{x}_i, \tilde{x}_j)$ is representable
by its fuzzy set. The relation $\text{TTend}$ is antireflexive,
antisymmetric and transitive:

$$ \forall \tilde{x} \in \tilde{X} \, \text{TTend} (\tilde{x}, \tilde{x}) = 0, $$

$$ \forall \tilde{x}, \tilde{y} \in \tilde{X} \, \text{TTend} (\tilde{x}, \tilde{y}) \land \text{TTend} (\tilde{y}, \tilde{x}) = 0 $$

$$ \forall \tilde{x}, \tilde{y}, \tilde{z} \in \tilde{X} \, \text{TTend} (\tilde{x}, \tilde{z}) > \text{TTend} (\tilde{x}, \tilde{y}) \land \text{TTend} (\tilde{y}, \tilde{z}) $$

The indicated properties of the relation $\text{TTend}$ permit to classify it as a relation of difference, with it the aggregate of
all possible evaluations $A = \{\alpha_0\}$ forms the fuzzy scale $S_a$

$$ = \langle \text{Name}_{\_} \text{RTend}, \, A, \, \tilde{X}, \, G_a, \, P_i \rangle. $$

Thus, the ACL-scale $S_e$ for determination of linguistic evaluations is the two-level scale. At the first level of
hierarchy from its universal set the ACL-scale $S_e$ permits to
determine linguistic evaluations $\tilde{x}_i$ for values $x \in X$ which characterize their qualitative aspects. Such linguistic
evaluations relate to the class of absolute LE. At the second
level of hierarchy for values $\tilde{x}_i$ and $\tilde{x}_j$ – linguistic
evaluations of their changes $(\nu_0, \alpha_0)$ which characterize qualitative aspects of differences or "difference of the first
order" by scales $S_a, \, S_e$. Such linguistic evaluations are related
to comparative LE.

Let us consider peculiarities of ACL-scales. The offered
linguistic ACL-scale $S_e$ is related to the class of fuzzy
evaluation scales which enter into the class of ordinal scales,
difference and the degree of difference can be additionally
evaluated in it. This property permits to consider the linguistic
evaluation ACL-scale $S_e$ as quasi-interval and to determine
"evaluation" and "computing" operations for it.

Let us introduce the following "evaluation" operations of the
ACL-scale $S_e$ generating linguistic evaluations:

1. The operation of determination of the absolute
linguistic evaluation $\tilde{x}_i$ by the value of characteristic
of the object $x_i$ being evaluated

$$ \tilde{x}_i = \text{Fuzzy}(x_i), \, x_i \in B, \, \tilde{x}_i \in \tilde{X}. $$

2. The operation of determination of the value of
characteristic of the object $x_i$ being evaluated by the
absolute linguistic evaluation $\tilde{x}_i$

$$ x_i = \text{DeFuzzy}(\tilde{x}_i), \, x_i \in B, \, \tilde{x}_i \in \tilde{X}. $$

3. The operation of determination of the type of
difference (comparative linguistic evaluation)

$$ \nu_0 = \text{TTend}(\tilde{x}_i, \tilde{x}_j), \, \tilde{x}_i \in \tilde{X}, \, \tilde{x}_j \in \tilde{X}. $$

The operation $\text{TTend}$ is non-commutative.

4. The operation of determination of intensity of
difference (comparative linguistic evaluation)

$$ \alpha_0 = \text{RTend}(\tilde{x}_i, \tilde{x}_j), \, \tilde{x}_i \in \tilde{X}, \, \tilde{x}_j \in \tilde{X}. $$

The operation $\text{RTend}$ is commutative.

Let us determine the aggregate of "computing" operations of
the ACL-scale for generated linguistic evaluations:

1. Computing the absolute linguistic evaluation

$$ \tilde{x}_i = \text{Comp}(\tilde{x}_i, \nu_0, \alpha_0). $$

2. The difference of intensities of differences

$$ \alpha_0 = \text{Diff}(\alpha_i, \alpha_j) $$
3. The union of intensities of the difference
\[ \alpha_j = \text{Union}(\alpha_i, \alpha_j). \]

4. The intersection of intensities of differences
\[ \alpha_j = \text{Inter}(\alpha_i, \alpha_j). \]

Operations \textit{Diff}, \textit{Union}, \textit{Inter} are commutative, associative, bounded.

Linguistic evaluations received by the indicated linguistic ACL-scale will be used in the next part as semantic-dependent solution of the problem of summarization of FTS in terms of fuzzy tendencies.

IV. APPLICATION OF ELEMENTARY TENDENCY MODEL FOR SUMMARIZATION OF FUZZY TIME SERIES

Let us consider the application of the offered ACL-scale in solution of the problem of summarization of FTS as the problem of identification of its general fuzzy tendency. For this purpose, let us design the hierarchical granular model for the initial time series \( X = \{x_i\}, i \in [1,n] \), \( n \) – is the quantity of members of the series. Let us introduce four levels of granulation forming of which corresponds to the bottom-up approach. The zero level of granules will be represented by fuzzy labels of the initial TS. For forming the granules of the zero level, let us use the "evaluation" operation of the ACL-scale: \( \tilde{X}_i = \text{Fuzzy}(x_i) \).

Let us compare fuzzy tendencies of FTS and information granules which have structural commonness. Let us define the operation of granulation of the first level in the form of the functional \( ETend \) forming the granules of elementary fuzzy tendencies: \( \tau_i = ETend(\tilde{X}_i, \tilde{X}_{i+1}) \), \( i \in [1,n-1] \), \( n \) – is the quantity of members of FTS.

The functional \( ETend \) generates granules of elementary fuzzy tendencies on the basis of "evaluation" operations \( TTend \) and \( RTend \) of the ACL-scale and can be realized in the subsystem of fuzzy deduction with rules of the following form:

\[ R_i := \text{IF } x_i \text{ is } A_{i,1} \text{ AND } x_{i+1} \text{ is } A_{i,2} \text{ THEN } v_i \text{ is } B_i \text{ AND } a_i \text{ is } N_i \]

The semantics of rules of realization of the functional \( ETend \) is represented in the following table.

<table>
<thead>
<tr>
<th>TABLE I.</th>
<th>TABLE OF RULES</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of the rule</td>
<td>( \tilde{X}_i )</td>
</tr>
<tr>
<td>1.</td>
<td>Sat</td>
</tr>
<tr>
<td>2.</td>
<td>Go</td>
</tr>
<tr>
<td>3.</td>
<td>Ex</td>
</tr>
<tr>
<td>4.</td>
<td>Bad</td>
</tr>
<tr>
<td>5.</td>
<td>Ze</td>
</tr>
<tr>
<td>6.</td>
<td>Ze</td>
</tr>
<tr>
<td>7.</td>
<td>Ze</td>
</tr>
<tr>
<td>8.</td>
<td>Ze</td>
</tr>
<tr>
<td>9.</td>
<td>Ze</td>
</tr>
<tr>
<td>10.</td>
<td>Bad</td>
</tr>
<tr>
<td>11.</td>
<td>Bad</td>
</tr>
<tr>
<td>12.</td>
<td>Bad</td>
</tr>
<tr>
<td>13.</td>
<td>Bad</td>
</tr>
<tr>
<td>14.</td>
<td>Sat</td>
</tr>
<tr>
<td>15.</td>
<td>Sat</td>
</tr>
<tr>
<td>16.</td>
<td>Sat</td>
</tr>
<tr>
<td>17.</td>
<td>Sat</td>
</tr>
<tr>
<td>18.</td>
<td>Go</td>
</tr>
<tr>
<td>19.</td>
<td>Go</td>
</tr>
<tr>
<td>20.</td>
<td>Go</td>
</tr>
<tr>
<td>21.</td>
<td>Go</td>
</tr>
<tr>
<td>22.</td>
<td>Ex</td>
</tr>
<tr>
<td>23.</td>
<td>Ex</td>
</tr>
<tr>
<td>24.</td>
<td>Ex</td>
</tr>
<tr>
<td>25.</td>
<td>Ex</td>
</tr>
</tbody>
</table>

The following abridgements were used in this table: Ze(Zero), Bad(Bad), Sat(Satisfactory), Go(Good), Ex(Excellent) for fuzzy values of FTS; Inc(INCREASE), Dec(DECREASE), St(STABILITY) for values of types of changes; Bt(Big), M(Medium), Sm(Small) and modifiers, such as Ve(Very), St(Significantly), No(No) for values of intensity of changes.

Let us define the operation of granulation of the second level in the form of the functional \( STend \) forming the granules of local fuzzy tendencies: \( \tau_j = STend(\tau_i, \tau_j) \), where \( \tau_i, \tau_j \) are granules of the first level.

The introduced functional \( STend \) is computed as the result of the union of one-type elementary tendencies on the base of the "computing" operation \( \text{Union} \) of the ACL-scale. Then the union \( \tau_j = STend(\tau_i, \tau_j) \) is the such fuzzy tendency for which \( v_j = v_i, \alpha_j = \text{Union}(\alpha_i, \alpha_j), \mu_j = \mu_i \cup \mu_j \), the duration \( \Delta_j = \Delta_i + \Delta_m \).

The operation of union of one-type tendencies defines the granules of the second level.

The generalized form of rules of granulation of the second level on the basis of the functional \( STend \) has the form:

\[ R_{i1} := \text{IF } v_i \text{ is Inc THEN IF } a_i \text{ is } A_{i,1} \text{ AND } a_{i+1} \text{ is } A_{i,2} \text{ THEN } TInc \text{ is } B_i \]

\[ R_{i2} := \text{IF } v_i \text{ is Dec THEN IF } a_i \text{ is } A_{i,1} \text{ AND } a_{i+1} \text{ is } A_{i,2} \text{ THEN } TDe \text{ is } B_i \]

The semantics of rules of granulation of the second level is given below:
TABLE 2. TABLE OF RULES LEVEL 2

<table>
<thead>
<tr>
<th></th>
<th>( T_{\text{Inc}} )</th>
<th>( T_{\text{Dec}} )</th>
<th>( T_{\text{end}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bi</td>
<td>Me</td>
<td>Inc</td>
</tr>
<tr>
<td>2</td>
<td>VeSm</td>
<td>Sm</td>
<td>Dec</td>
</tr>
<tr>
<td>4</td>
<td>Me</td>
<td>Bi</td>
<td>Dec</td>
</tr>
<tr>
<td>5</td>
<td>Bi</td>
<td>Sm</td>
<td>Inc</td>
</tr>
<tr>
<td>6</td>
<td>Me</td>
<td>Sm</td>
<td>Inc</td>
</tr>
<tr>
<td>7</td>
<td>Sm</td>
<td>Me</td>
<td>Dec</td>
</tr>
</tbody>
</table>

On the basis of introduced functionals we defined the procedure of summarization of TS as the procedure of identification of the fuzzy tendency \( T_{\text{end}} \). This procedure is the sequential generation of information granules which model TS at different abstract levels. The result of the procedure of summarization of FTS is the granule of the general fuzzy tendency which is the convolution of elementary tendencies into the linguistic evaluation of behaviour of a FTS:

\[
ET_{\text{end}}(\tilde{x}_i, \tilde{x}_{i+1}) \rightarrow ST_{\text{end}}(\tau_i, \tau_s) \rightarrow GT_{\text{end}}(\tau_i, \tau_s) .
\]

The offered approach to the solution of the problem of summarization of FTS on the basis of fuzzy tendency model and granular computing was realized as software in the system generating artificial time series with noise. On figures 1, 2 examples of execution of the procedure of summarization of the artificial TS and its results are presented.

REFERENCES


